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Thesis

A BETA-RAY SPECTROMETER WITH ELECTROSTATIC FOCUSSING

by

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Introduction

From the time that Becquerel first noted that a portion of the radiation from a radioactive source called beta radiation could be strongly deflected by a perpendicular magnetic field physicists have worked at devising methods and means for measuring the intensities and energies of the particles composing this part of the radiation which were early identified as electrons. Various methods have been used for this purpose.

A device called the spinthariscope was used by Regener to determine the total beta emission from a radioactive source. A tank with a source placed inside was evacuated, and a zinc sulphide screen was placed in an opening at one end of the tank so that it could be viewed through a microscope. The particles striking the screen would form temporary illuminated spots that could be counted if they were not to numerous. Since the solid angle subtended at the source was known, the total emission could be determined. However, because of the small mass of the electron each of the scintillations was probably caused by several electrons striking close together.

Another principle that has been used extensively for the determination of the intensity of a source as well as for detecting radiation in general is the fact that radioactive particles will cause ionization in a gas which will cause the gas to become electrically conducting. A radioactive source placed near a charged electroscope will cause the leaves to collapse. The activity of the source is inversely proportional to the time required for the leaves of the electroscope to collapse. Modifications of this method have been used by various investigators.

The ionization chamber, which is a modification of the electrometer, has been used very satisfactorily for detecting and measuring the activity of a source. A cylinder of screen wire is placed inside a chamber which will hold pressures up to several atmospheres. The chamber is electrically grounded, while the screen is maintained at a high electric potential. A rod in the center of the cylinder is insulated from the screen and chamber and is connected through a sulphur bushing in the chamber to a quartz fiber which has been made electrically conducting by a metal coating. The wire is under slight tension between two oppositely charged plates where it is viewed through a microscope provided with a calibrated scale. As the central rod acquires

a charge it is conducted to the quartz fiber, and the fiber will be deflected away from the charged plate of like charge as that on the wire. The amount of deflection is proportional to the charge on the wire which in turn is proportional to the activity of the radioactive source. The sensitivity of the apparatus can be changed by changing the potential differences on the plates. By replacing the quartz fiber and charged plates with a vacuum tube amplifying circuit the ionization current in the chamber could be determined which is proportional to the activity of the source. Refinements of this method led to the Geiger-Mueller counter which is used universally today for the detection of radioactivity and for the measurement of the intensity of a source.

The methods described so far measured the intensity of a particular source, but they didn't give any information concerning the energies of the electrons in the beta radiation. In 1911, the Wilson Cloud Chamber[1], named after the inventor C.T.R. Wilson, was demonstrated. By the use of this instrument it was possible to see and photograph the path of an individual particle. Water or alcohol or any liquid whose vapors have negative specific heats are placed in the bottom of a chamber which is equipped with bellows and an

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[1] C.T.R. Wilson, Proc. Roy. Soc. A, 85, 285 (1911).

expansion chamber so that the vapors above the liquid can be compressed and rapidly expanded. In a supersaturated vapor condensing droplets will form on any electrically charged particles present. With vapors of negative specific heats, a supersaturated vapor can be produced by suddenly increasing the volume in which the saturated vapors are contained. Vapors of positive specific heats can also be used, but the volume containing the saturated vapors must be suddenly reduced. Any radioactive particle passing through the vapor will produce ions all along its path. If a particle passes through just as a supersaturated vapor is formed, each of the ions formed by the particle will have droplets formed on it which can be seen and photographed thus outlining the path which the particle followed through the chamber. An electric field is applied to the chamber until just before the track is to be observed to remove any ions that may have diffused through the chamber. As soon as expansion is completed, the chamber is lighted and a photograph of the tracks made.

A magnetic field applied perpendicular to the floor of the chamber causes the particles to be deflected at right angles to the direction of their motion and to the magnetic field thus causing their

paths to be circular; the direction of deflection depending on the charge on the particle. The particles can thus be identified by the direction of their motion, and from their radii of curvature their momenta and thus their energies can be determined. The results obtained by this method are sometimes lacking in exactness because of scattering effects in the chamber.

The beta rays were found to be composed of electrons of energies ranging from a maximum value down to zero. In order to determine the number of particles in each energy group beta ray spectrometers of various types were developed.

In the first type of spectrometer devised, a beam of electrons from a beta emitting source passed through a defining slit into a uniform magnetic field perpendicular to the direction of the beam of electrons. The beam was spread into a spectrum by the magnetic field due to the fact that the slower, less energetic electrons will have a smaller radius of curvature than the more energetic ones. This spectrum was registered on a photographic plate. The radius of curvature and hence the momentum and energy of the particles could be determined. The intensities of the various energy groups could be determined by the relative darkening of the photographic plate in the areas where

the various energy groups landed. This method was not toosatisfactory, however, because the source was finite instead of being a point source. Thus there was an overlapping of energy groups on the photgraphic plates since particles of the same energy could have varying directions on passing through the slit which changed their position on the photographic plate.

This method was first used by Danysz[1] who later developed the magnetic semicircular focusing spectrometer. This later type was improved and used by Robinson and Rutherford[2] and is still used with variations in design.

The source is placed at a point slightly below a defining slit. A photographic plate is placed on a lead block 180° from the slit. The chamber is evacuated and placed in a uniform magnetic field directed perpendicular to the plane of the source and photographic plate. Particles emerging from the slit having the same velocities will describe circles of equal diameters. If the slit is not toowide, all particles with the same energies will land almost on the same spot on the photographic plate. The particles passing through the center of the slit will actually land a little farther from the slit than those passing on either side. This will give a sharp outer edge with a

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1. J. Danysz, Le Radium, 9,1 (1912); 10,4 (1913).
2. H. Robinson & E. Rutherford, Phil. Mag.,26, 717 (1913).

more diffuse inner edge. Those particles of different energies will land at different distances from the slit. The intensities of the various energy groups are determined by measuring the relative darkening of the photographic plate with a densitometer.

In later improvements of this type of spectrometer, a slit is cut in the bottom of the tank at 180° from the slit, and a window with low electron absorbing properties is placed in the slit to make the tank air tight. The electrons emerging from the slit are counted by a Geiger tube and counting circuit. The magnetic field is varied in this case to bring particles of varying energies to the slit.

Another type of spectrometer uses the principle of the electron lens for measuring the energy distribution. A source is placed at one end on the axis of an evacuated cylinder around which a solenoidal wound conductor is placed. A Geiger counter is placed at the other end on the axis. Lead baffles are placed between the source and Geiger tube. When current flows through the coils, an almost uniform magnetic field will be set up parallel to the axis. Since baffles are placed along the axis, only those particles starting off at an angle to the axis will reach the detector.

Since particles starting from the axis at an angle cross the magnetic lines of force at an angle, they will tend to execute helical paths. For a particular magnetic field, particles with the right velocity will complete a circle in the time that it takes it to pass from the source to the Geiger tube and will be back on the axis at the time it reaches the Geiger tube. All particles of different velocities will not be back on the axis in the same time and will not be registered by the counter. By varying the current through the coil and hence changing the magnetic field, a different energy group will arrive at the detector. The momentum of the particles varies linearly with the current in the coils. Thus the spectrometer can be calibrated for various energies by calibrating current against known energies. Spectrometers using the same principle except using short bunched coils such as are used in the electron microscope have been constructed by Klemperer[1] and Deutsch[2].

Hughes and Rojansky[3] first determined that electrons of the same energy could be focused by the use of a radial, first power, electrostatic field. They calculated that for an angle of 127°17' between the source and the detector, the electrons of the same

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1. O. Klemperer, Phil. Mag., 20,545 (1935).
2. M. Deutsch, Phys. Rev., 59, 684 (1941).
3. M. Deutsch, L.G.Elliot, & R.D. Evans, Rev.Sci.Inst., 15,178 (1944) .
3. A.L. Hughes & V.Rojansky, Phys.Rev.,34,284 (1929).

energy passing through the entrance slit will refocus at the exit slit and pass into the detector. Two concentric grids set up the inverse first power, radial field. There is a linear relationship between the energy of the particles and the voltage difference between the electrodes. Spectrometers of this type have been used by Backus[1] and Rogers.[2]

The methods outlined above give the basic principles in use in beta-ray spectrometers. Other modifications of these principles are possible to be incorporated in future devices. The design of the spectrometer proposed in this paper uses some of the features of several of the instruments described. In this spectrometer, the electrons emitted by the radioactive source are accelerated and focused by a special type of electrostatic lens. The source and lens are shielded by iron to prevent magnetic fields from interferring with the focussing action of the lens.

After the particles leave the lens, they pass through an opening in the iron shield into a uniform magnetic field perpendicular to the motion of the electrons. The electrons move in a radial path, the radius of the path being determined by the velocity of the electrons. Upon passing through an exit window, the particles are detected by a Geiger counter.

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[1] J. Backus, Phys.Rev., 68, 59 (1945).

[2] F. T. Rogers, Rev.Sci.Inst. 8,22,(1937); 11,19 (1940); F.T. Rogers & C.W. Harton, Rev.Sci.Inst.,14,216(1943).

The purpose of the changes in design of this spectrometer from other types is to make possible the more accurate study of the lower end of the beta spectrum. Difficulties are encountered in the study of low energy beta particles with instruments in use at the present time because many of the particles are absorbed in the exit windows and Geiger tube windows due to their low energies. With the electrostatic lens to be used with this spectrometer all particles will be uniformly accelerated so that the same relative position of the energy groups in the spectrum will be maintained, but the low energy groups will be given sufficient energy to pass easily through the exit and counter windows.

Another feature which could be made use of with this instrument is the possibility of making coincidence counts of various definite known energy groups of electrons from the same radioactive source thereby gaining furthur information on the subject of beta decay.

PART I

Lens Design

Section A: Theory

When an electron enters a space of which the electrostatic potential ϕ is known at every point, a force will be exerted on the electron equal to $e\frac{\partial \phi}{\partial q}$, where e is the charge on the electron and q is the generalized spatial coordinate. This force will be directed along the electric field vector which extends perpendicular to an equipotential line. If the electrostatic field in the space is non-uniform and so shaped that the electron in its motion is continually accelerated in the same direction, this space can be used as an electrostatic lens for focussing electrons in a manner analogous to a light lens. Such a field can be set up by applying a potential difference between two electrodes of a special type.

Before it is possible to calculate the path of an electron through the field, it is necessary to know the potential distribution between the electrodes. To do this it is necessary to solve the Laplace equation with boundary conditions dictated by the shapes of the electrodes and the potentials applied. The theory

given here of the solution of this problem and the tracing of an electron through the field follows, mainly, the discussion of Zworykin and Morton in their book Television[1], chapters 3 and 4, and Maloff and Epstein in their article, "Theory of the Electron Gun".[2]

The solution of the Laplace equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad (1)$$

will give the potential as a function of the spatial coordinates. The electrostatic field can then be determined by differentiating the solution with respect to the coordinates. Then

$$E_x = -\frac{\partial\phi}{\partial x}(x,y,z) \quad ;$$

$$E_y = -\frac{\partial\phi}{\partial y}(x,y,z) \quad ;$$

$$E_z = -\frac{\partial\phi}{\partial z}(x,y,z) \quad .$$

It is then seen from the original equation that the field must satisfy the equation

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \ . \qquad (2)$$

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1. Television. The Electronics of Image Transmission, V.K. Zworykin and G.A. Morton; John Wiley and Sons, Inc., publisher.
2. I.G. Maloff and D.W. Epstein, Theory of the Electron Gun. Proc.I.R.E., Dec.1934, Vol. 22, No. 12.

No general solution for the Laplace equation exists. An analytic solution can be obtained for only certain simple electrode configurations such as two infinite parallel plates, concentric spheres, concentric cylinders etc., and for certain slightly more complex configurations but then only by laborious approximation schemes such as series expansion or numerical integration.

Since most of the electrodes used in electron lenses are cylinders or apertures, we can make use of the fact that cylindrical symmetry exists in these electrodes, and the Laplace equation can then be expressed to best advantage in cylindrical coordinates, z, ϕ, r. It then becomes

$$\frac{\partial^2 \phi}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) = 0 \quad . \qquad (3)$$

The general method of solution in this type of problem is to try separation. Assume

$$\phi(z,\theta,r) = F(r)\, G(\theta)\, Z(z) \quad . \qquad (4)$$

By substituting for ϕ in equation (3) the equation reduces to

$$GZ\frac{\partial^2 F}{\partial r^2} + \frac{GZ}{r}\frac{\partial F}{\partial r} + \frac{FZ}{r^2}\frac{\partial^2 G}{\partial \theta^2} + FG\frac{\partial^2 Z}{\partial z^2} = 0. \qquad (5)$$

After separating variables the equation becomes

$$\frac{1}{F}\frac{d^2F}{dr^2} + \frac{1}{Fr}\frac{dF}{dr} + \frac{1}{Gr^2}\frac{d^2G}{d\theta^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0 \quad . \qquad (6)$$

This can be separated into two parts, and the part in Z can be solved directly. Then

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -k^2 \quad , \qquad (7)$$

where k^2 is the separation parameter. The solution to this equation is

$$Z = C_1 e^{\pm ikz} \quad . \qquad (8)$$

The remaining equation then becomes

$$\frac{r^2}{F}\frac{d^2F}{dr^2} + \frac{r}{F}\frac{dF}{dr} + \frac{1}{G}\frac{d^2G}{d\theta^2} - r^2k^2 = 0 \quad . \qquad (9)$$

This equation again separates into two parts,

$$\frac{1}{G}\frac{d^2G}{d\theta^2} = -\ell^2 \qquad (10)$$

and

$$\frac{r^2}{F}\frac{d^2F}{dr^2} + \frac{r}{F}\frac{dF}{dr} - (\ell^2 + r^2k^2) = 0 \quad . \qquad (11)$$

The solution of equation (10) is

$$G = C_2 e^{\pm i\ell\theta} \quad . \qquad (12)$$

Upon substitution of $x = i\,k\,r$ in equation (11) this becomes Bessel's equation

$$x^2\frac{d^2F}{dx^2} + x\frac{dF}{dx} + (x^2 - \ell^2)\,F = 0 \quad . \qquad (13)$$

The solution to equation (11) is then

$$F = C_3 J_\ell (ikr) + C_4 N_\ell (ikr) \quad . \qquad (14)$$

With cylindrical symmetry $\frac{\partial^2 \phi}{\partial \theta^2} = 0$ in our lens, and, therefore, from equation (10) $\ell = 0$. The solution, equation (14), then becomes

$$F = C_3 J_0 (ikr) + C_4 N_0 (ikr) \quad , \qquad (15)$$

where J_0 is the Bessel function and N_0 is the Neuman function. Combining results (8) and (15) we get

$$\phi(r,z) = \sum_{k} C_k F_k (r) Z_k (z) \quad . \qquad (16)$$

Since k does not necessarily have discrete values, we may replace the summation by integration, and the solution becomes

$$\phi(r,z) = \int C(k) F(r,k) Z(z,k) dk . \quad (17)$$

If the boundary conditions are not too complex, this integration can be performed, but even then resort must usually be made to laborious approximation methods. The axial potential and its first two derivatives have been worked out from equation (17) for two coaxial cylinders of equal diameters. They are here reproduced from Television[1], page 111.

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[1] Loc. cited.

$$\phi(0,z) = \frac{1}{\pi}\int_0^\infty (\phi_2 - \phi_1)\,\frac{\sin kz}{kJ_o(ik)}\,dk + \frac{\phi_1 + \phi_2}{2}, \qquad (18)$$

$$\phi'(0,z) = \frac{1}{\pi}\int_0^\infty (\phi_2 - \phi_1)\,\frac{\cos kz}{J_o(ik)}\,dk, \qquad (19)$$

$$\phi''(0,z) = -\frac{1}{\pi}\int_0^\infty (\phi_2 - \phi_1)\,\frac{k\sin kz}{J_o(ik)}\,dk. \qquad (20)$$

These integrals must be evaluated by approximation methods, and the numerical values of the potentials obtained are used to calculate the electron paths through the lens.

With a change of the boundary conditions, the problem usually becomes so involved that in most cases it is impossible to find an analytic solution even by approximation methods. Since the potential distribution in a lens with two coaxial cylinders does not have the proper shape to give all electrons an equal acceleration in passing through the lens, and a more complex type of configuration was thus necessary to give the results required in the lens under consideration in this work, it was decided to determine the potential distribution by experimental means. The experimental methods employed and results obtained will be described in Section B of this paper.

In order to determine the characteristics of the lens it is necessary to trace the path of an electron through the lens. We will consider the case of a paraxial electron, that is, an electron moving parallel to the axis of the lens or where the path makes a very small angle with the axis of the lens.

The value of the radius of curvature of an equipotential surface through a point on the axis is given by

$$R = \frac{2\frac{d\phi}{dz}}{\frac{d^2\phi}{dz^2}} \quad . \qquad (21)$$

There is a simple relationship between radial and axial gradients.

$$\frac{\partial\phi}{\partial r} : r = \frac{\partial\phi}{\partial z} : R \qquad (22)$$

or

$$\frac{\partial\phi}{\partial r} = \frac{r\frac{\partial\phi}{\partial z}}{R} = r\,\frac{\frac{\partial\phi}{\partial z}\,\frac{\partial^2\phi}{\partial z^2}}{2\frac{\partial\phi}{\partial z}} = \frac{1}{2}\,r\,\frac{\partial^2\phi}{\partial z^2} \qquad (23)$$

where r is the radial distance of the paraxial electron from the axis. The radial acceleration of an electron is then

$$\frac{d^2r}{dt^2} = \frac{1}{2}\,\frac{e}{m}\,r\,\frac{d^2\phi}{dz^2} \quad , \qquad (24)$$

but

$$\frac{dz}{dt} = \sqrt{2\frac{e}{m}\phi} \quad . \qquad (25)$$

Then

$$\frac{d^2r}{dt^2} = \frac{dz}{dt}\frac{d}{dz}\left(\frac{dr}{dz}\frac{dz}{dt}\right) = 2\frac{e}{m}\phi\frac{d^2r}{dz^2} - \frac{e}{m}\frac{dr}{dz}\frac{d\phi}{dz} , \qquad (26)$$

$$\frac{d^2r}{dz^2} = \frac{1}{4}\, r\,\frac{1}{\phi}\frac{d^2\phi}{dz^2} - \frac{1}{2}\frac{1}{\phi}\frac{dr}{dz}\frac{d\phi}{dz} , \qquad (27)$$

and $$r = \iint_z \frac{1}{4}\, r\,\frac{1}{\phi}\frac{d^2\phi}{dz^2}\, dzdz - \iint_z \frac{1}{2}\frac{1}{v}\frac{dr}{dz}\frac{d\phi}{dz}\, dzdz . \qquad (28)$$

An examination of the relationship in equation (28) will show that the path of the electron depends only on r, z and relative ϕ which means it depends on the geometry of the field and the focussing ratio only. The relationship also shows that an electron moving with a velocity corresponding to a drop through a voltage ϕ_1, and going through an electrostatic field caused by a potential difference $\phi_2 - \phi_1$, follows a path independent of the values of ϕ_1, and ϕ_2 but is determined by the ratio $\frac{\phi_2}{\phi_1}$.

After the potential distribution has been determined, the path of an electron travelling parallel to the axis in object space is traced through the lens until it crosses the axis. This point locates the second focal point of the lens. By extending a line tangent to the ray back until it intersects the incident

parallel ray, the position of the second principal plane is located. The position of the first focal point and first principal plane are similarly determined by considering an incident ray parallel to the axis in image space. With the information obtained, it is then possible to determine all the characteristics of the lens.

Section B

Experimental Procedure and Results

Since it was considered difficult in the extreme, if at all possible, to find the potential distribution in the lens by a mathematical solution of the Laplace equation, it was decided to map the field by means of the electrolytic plotting tank. The principle of the plotting tank makes use of the fact that the electrolyte used in the tank can be assumed to be an ohmic conductor so that the field strength at any point between the electrodes immersed in the liquid will be proportional to the current density.

The tank used for the plotting was built of wood and painted with water-proof varnish to make it water tight. The material of which the tank is constructed must be non-conducting so that the equipotential surfaces around the electrodes meet the walls perpendicularly, and thus the influence of the tank on the field is reduced. The tank used was two feet long, one and one half feet wide, and one foot deep. The liquid used as the electrolyte was water with a very small amount of copper sulphate added.

Since the electrodes used for the lens have mirror symmetry, and since the equipotential lines around electrodes cross a plane of symmetry at right angles, each electrode used was constructed to represent one half of each of the two lens electrodes cut through a plane of symmetry. These two models were immersed in the electolyte with the surface of the liquid coinciding with the plane of symmetry. The surface of the liquid is a perfect insulating plane so that no current can flow above the liquid when potentials are applied to the model electrodes. Thus the equipotentials must intersect the plane of symmetry at right angles because there is zero vertical current flow, and the field vector normal to the plane is zero.

Since the current flowing between the electrodes acts like an incompressible fluid, the current density and thus the field strength obey the equation of continuity, or, in other words, the potential throughout the electrolyte obeys the Laplace equation. The potential anywhere in the liquid is then proportional to the potential of a corresponding point in the actual lens electrode system.

The potential distribution over the plane of symmetry was measured by a small wire mounted so as to just break the surface and constrained to move in a

horizontal plane. The probe was carried at the end of a pantograph linkage so that the position and motion of the probe was transmitted directly to a recording pencil attached to the other end of the linkage where the motion of the probe was recorded on graph paper.

The shape of the electrodes was determined from the fact that since an equal acceleration of all electrons passing through the lens was desired, it was necessary to have a potential distribution of such a shape that the distance between any two successive equipotential surfaces measured along the field vector of the surface of highest potential is the same over a considerable portion of the lens. The electrode shape shown in figure (1) has equipotential surfaces that are spherical in shape. Thus it was decided to use electrode models of such a shape because it seemed possible to obtain the conditions desired with spherical equipotential surfaces. The dimensions of the models as originally constructed are given in figure 1. The

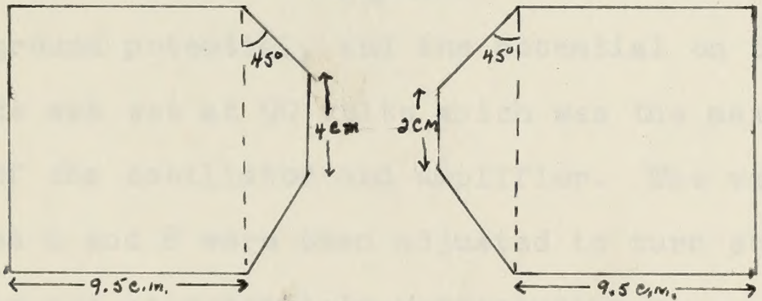


Figure 1.

reason why one of the apertures was made larger than the other is that most lenses of this type show less spherical aberration when the high potential electrode of the lens is smaller than the low potential electrode.

The experimental apparatus was arranged as in figure 2. The electrode with the larger aperture was set at ground potential, and the potential on the other electrode was set at 90 volts which was the maximum output of the oscillator and amplifier. The variable resistors A and B were then adjusted in turn so that the probe was set at potentials representing percentages of the potential difference between the electrodes from 5 percent to 95 percent. At each of the settings of the resistors, the probe was moved so that the current to it remained zero thus plotting the equipotential

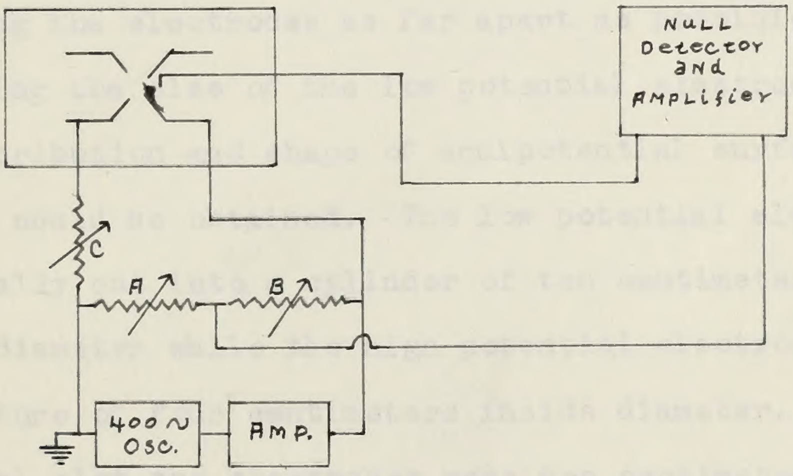


Figure 2.

surface of the percentage potential difference at which the probe was set.

With the original electrode sizes the equipotential surfaces obtained were too small for accurate determination of electron paths. Thus the apertures were cut to four and six centimeters in diameter, and plots were again made. It was found that by moving the electrodes as far apart as possible and increasing the size of the low potential electrode that the distribution and shape of equipotential surfaces desired could be obtained. The low potential electrode was finally cut into a cylinder of ten centimeters inside diameter while the high potential electrode was an aperture of four centimeters inside diameter. For the final plot the electrodes were ten centimeters apart. The lens configuration and equipotential plots are shown in figure 3.

The axial potential distribution is plotted as a function of the axial position in figure 4. The axial potential is plotted as a percentage of the potential difference between the two lenses. The position of the percentage equipotential lines is independent of the potential difference impressed on the electrodes so that this plot may be used to determine the potential

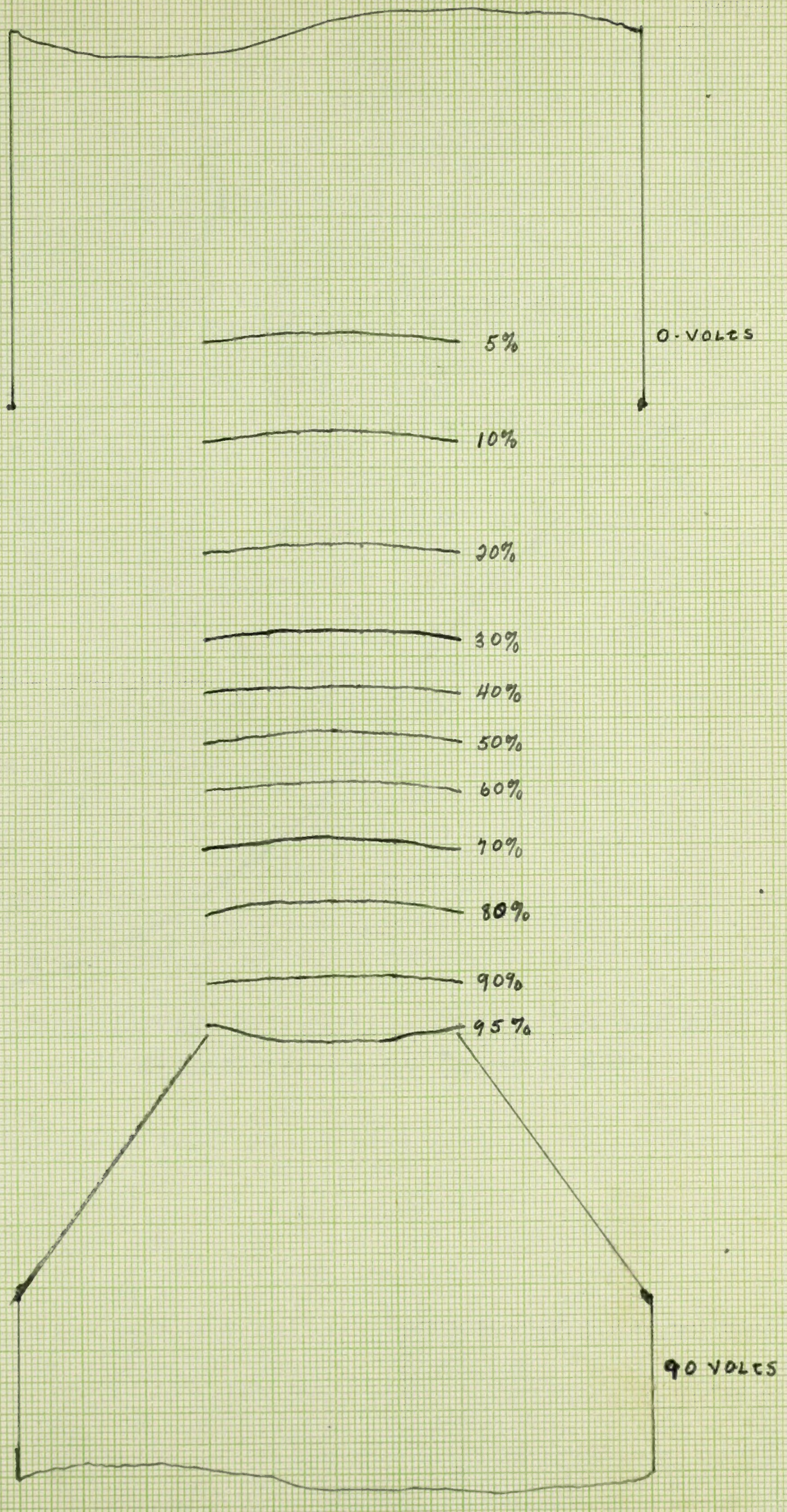


Figure 3.

Form No. 998–10 Millimeters to Centimeter
AMERICAN PAD & PAPER CO., HOLYOKE, MASS.

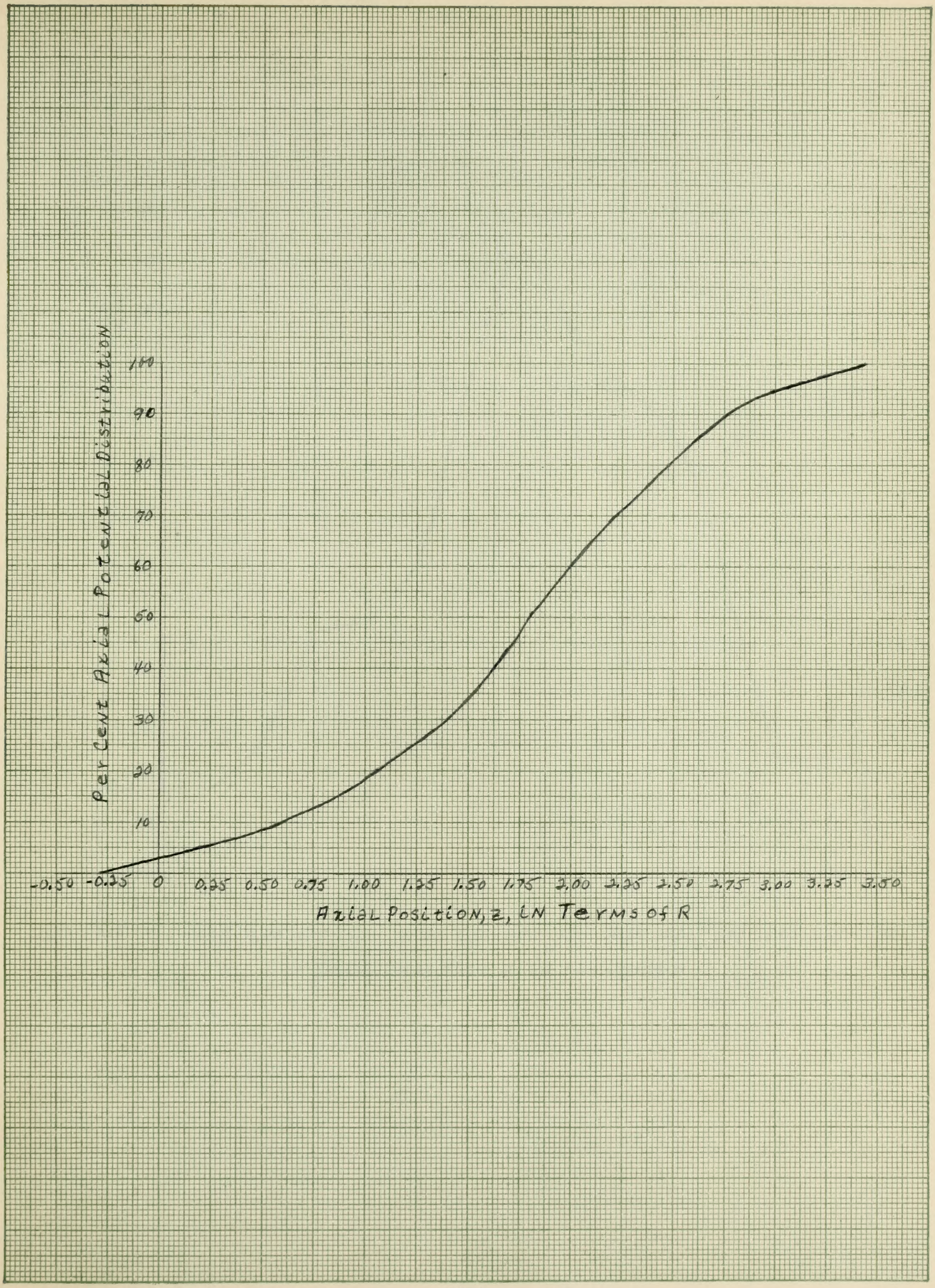
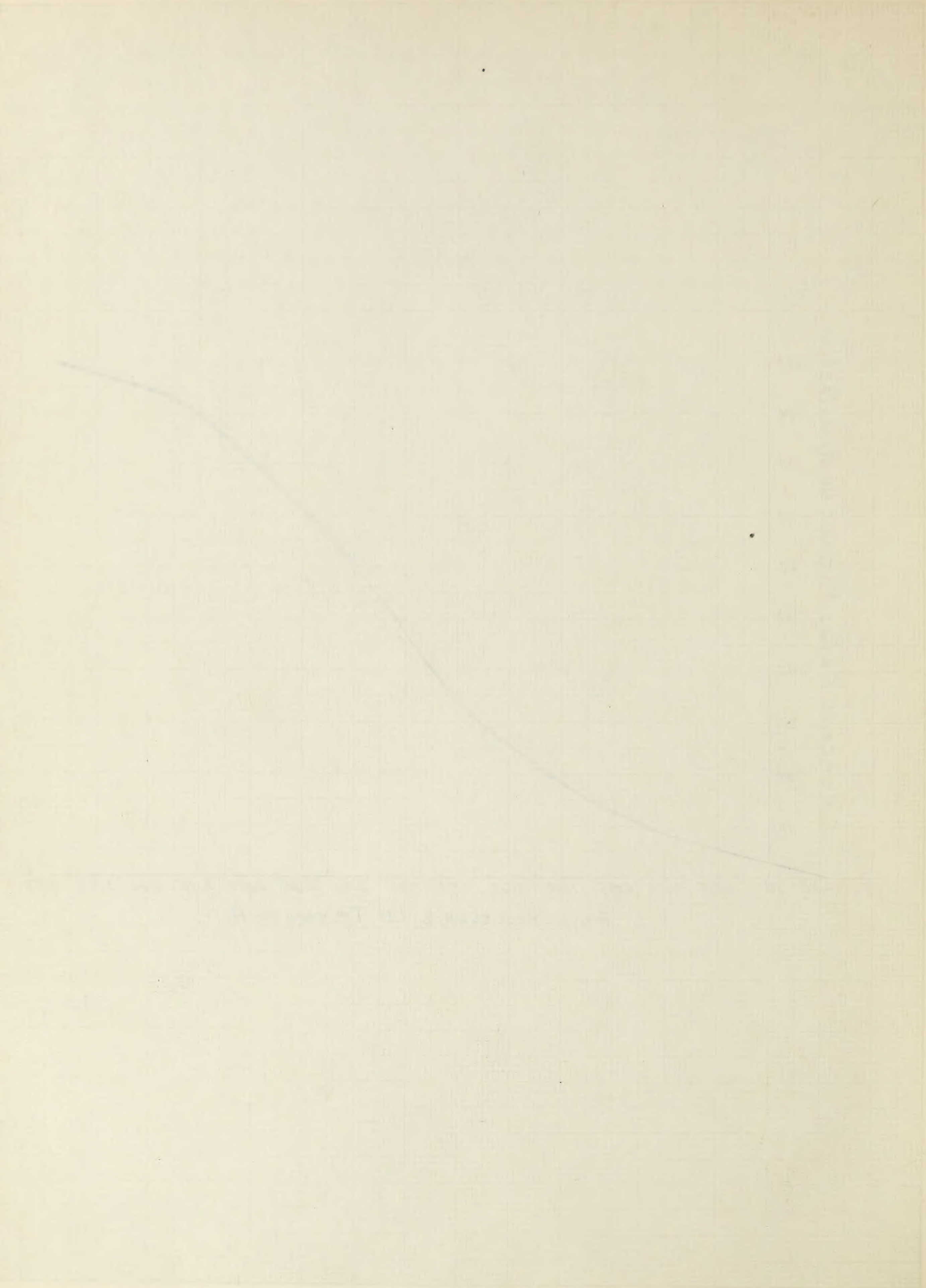


Figure 4.

10 Millimeters to Centimeter
AMERICAN PAD & PAPER CO., HOLYOKE, MASS.



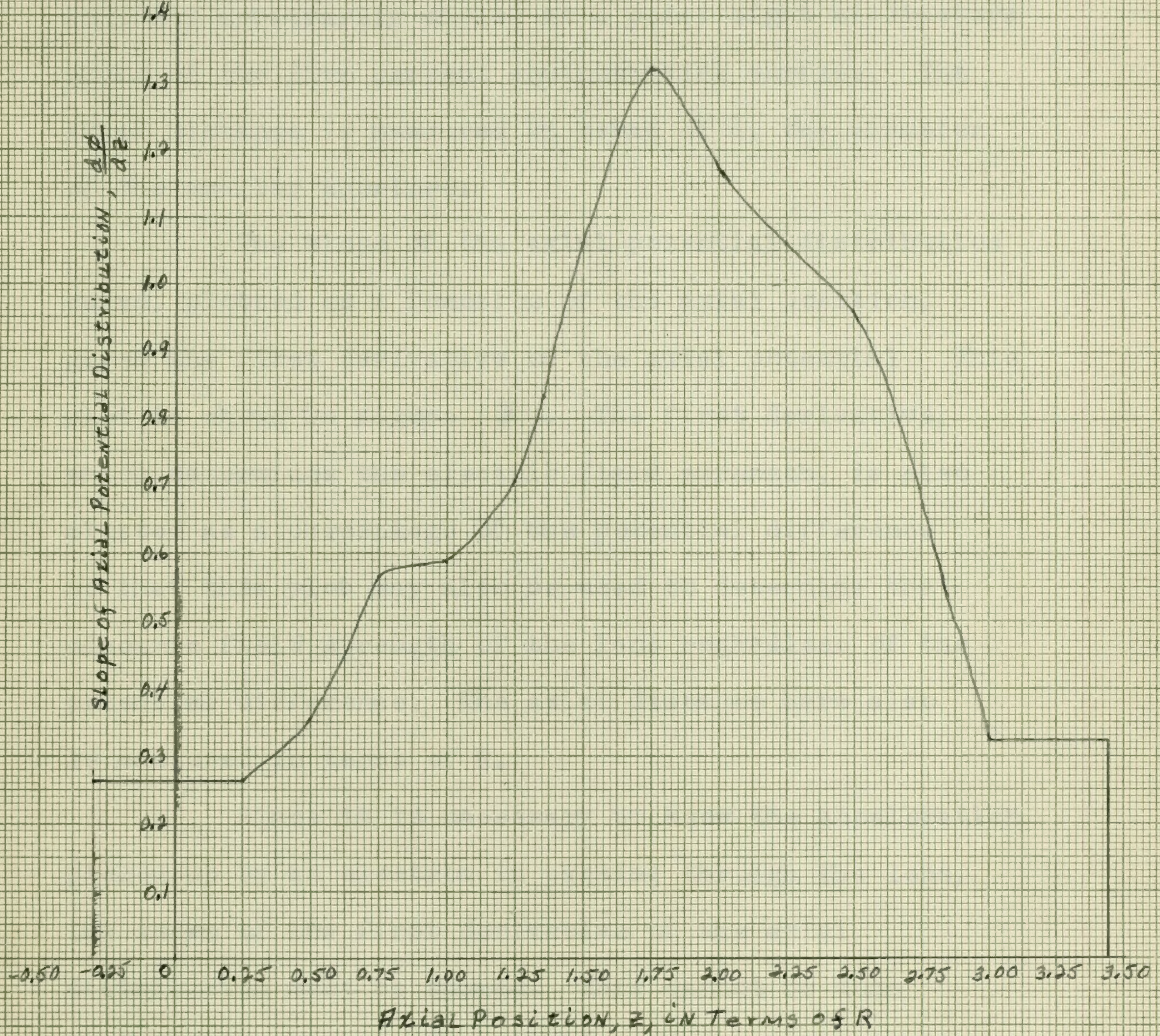


Figure 5.

AMERICAN PAD & PAPER CO., HOLYOKE, MASS.

at any point along the axis regardless of the potentials impressed on the electrodes. The zero point of the axis was arbitrarily chosen at a point eight millimeters ahead of the five percent equipotential surface. The axial distance is measured in terms of R, the diameter of the smaller aperture.

The slope of the axial potential distribution curve, plotted as a function of the axial position, gives the first derivative of the axial potential. This plot is shown in figure 5. The first derivative, which determines the electric field strength, is also plotted as a percentage of the potential difference so that this plot may also be used to determine the electric field strength of the lens regardless of the potential difference. The axial distance here also is measured in terms of R.

Since it is necessary to know the acceleration picked up by an electron in passing through the lens, the curve of the first derivative of the axial distribution must be summed. This cannot be done analytically because the equation of the curve isn't known. It can be done by finding the area under the curve. This was done by the use of a planimeter. To check the results of planimeter the ordinates of the

curve for each millimeter distance along the abscissa were summed, and, as a further check, the area under the curve was cut out of the graph paper and compared against a standard block of known area of the same piece of graph paper on a chemical balance. The latter method assumes, of course, that the graph paper is of uniform density throughout. The results obtained checked very closely. With the planimeter, the area found was 100.00 cm.^{2}, by the summation of the ordinates an area of 98.85 cm^2 was found, and with the chemical balance an area of 100.2 cm.^{2} was found. Thus the area found with the planimeter, being the most accurate, may be considered as the mean of the three determinations and will be used as the correct value.

In the plot of the first derivative one centimeter on the graph paper represents 0.1 volt/cm.. Thus 1 cm.^{2} of area = 0.1(volt/cm.^{2})cm. = 0.1 volt/cm. of field strength, so that the 100 cm.^{2} area found by the planimeter is equivalent to 10 volts/cm. of field strength. The final value of the field strength is, of course, dependent on the potential difference impressed between the electrodes; but when the potential is known, the field strength may be determined because the axial distribution is plotted as the percent potenial difference.

It is now necessary to trace the path of an electron through the lens so that the characteristics of the lens may be determined. There are several methods available for doing this. The methods used are described in *Television*[1], chapters 3 and 4. One method utilizes the fact that the path of the electron in passing through the region between two equipotential surfaces coincides approximately with the arc of a circle of radius $r = 2\frac{\phi}{E_r}$ tangent to the vector representing the velocity of the electron as it leaves the first equipotential surface. ϕ is the potential of the first equipotential surface. This method becomes difficult to use when the curvature of the path is small, as it is in our case, because the radii of the circles become awkwardly large.

The graphical method which can be used to advantage where the curvature of the path is small is the parabola method which will now be described. The accuracy of the two methods is about the same.

An electron moving in a uniform field with a velocity component at right angles to the field will follow a parabolic path. The tangent to a parabola at a point distant x from its vertex meets the axis at a distance 2x from the point of tangency. This is the principle involved in the parabolic method of ray

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[1] V. K. Zworykin and G. A. Morton, *Television* .

tracing. The electron is assumed to be at a point A on the first equipotential surface moving with a velocity v represented by a vector $\vec{v}$. A line representing the field vector $\vec{E}$ is drawn perpendicular to the equipotential, ϕ_1, at the point A which extends a distance d to the second equipotential ϕ_2. This line is drawn back a distance $\frac{2\phi_1 d}{\phi_2 - \phi_1}$ to a point B. A perpendicular is dropped from B onto the extension of $\vec{v}$ locating the point C. A line parallel to AB is then drawn back a distance d from C locating the point C'. A line drawn from C through A locates the position of the electron when it reaches ϕ_2.

The path of an electron through the lens is shown in figure 6. The electron was assumed to be moving parallel to the axis at a radial distance of 0.25R from the axis at the five percent equipotential surface. After the electron leaves the lens, it will continue in a straight line. The point at which the electron will cross the axis, according to the slope of the path after it leaves the lens, will be distant about 15.25R from the zero point of the axis. The total potential difference for the purpose of calculating $\frac{2\phi_1 d}{\phi_2 - \phi_1}$ was taken to be ten centimeters. The point where the electron crosses the axis determines the second

focal point of the lens.

Another method for tracing the path of an electron through the lens was also used to check the graphical method. This is the Gans method[1] which consists of representing the potential along the axis by straight-line segments, and applying the ray equation

$$\frac{d^2r}{dz^2} = -\frac{1}{2\phi}\frac{d\phi}{dz}\frac{dr}{dz} - \frac{1}{4\phi}\frac{d^2\phi}{dz^2}r ,$$

along the straight segments. The second derivative $\frac{d^2\phi}{dz^2}$ is zero over any straight segment. Then

$$\frac{d^2r}{dz^2} = -\frac{1}{2\phi}\frac{dr}{dz}\frac{d\phi}{dz} .$$

After integrating, this becomes

$$\frac{dr}{dz}\sqrt{\phi} = C$$

and

$$r = r_o + \frac{2C(\sqrt{\phi} - \sqrt{\phi_o})}{\frac{d\phi}{dz}}$$

where r_o and ϕ_o are the radial position and potential at the beginning of the segment.

At the point of intersection of two segments, $\frac{d^2\phi}{dz^2}$ is infinite, and we obtain the following equation over the transition:

$$\left(\frac{dr}{dz}\right)_2 - \left(\frac{dr}{dz}\right)_1 = -r\frac{\left(\frac{\partial\phi}{\partial z}\right)_2 \quad \left(\frac{\partial\phi}{\partial z}\right)_1}{4\phi} .$$

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[1] Zworykin and Morton, op. cited, pp. 106-110.

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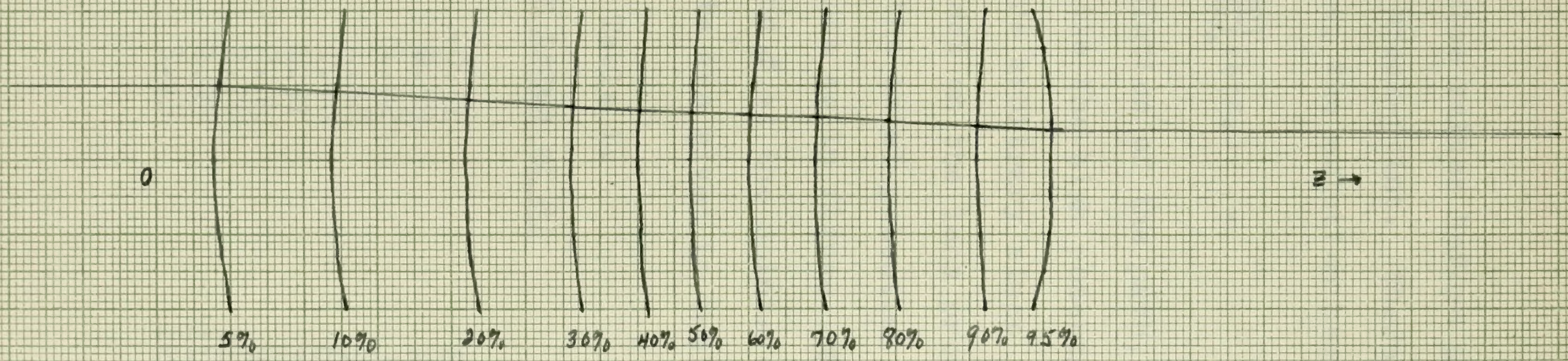

Figure 6.

The subscript 1 indicates values before the break-point, and 2 those after the break-point. Where the segment is parallel to the axis, the solution becomes

$$r = r_0 + \left(\frac{dr}{dz}\right)_0 (z - z_0) \quad .$$

In the calculation of the path, the low potential was assumed to be 2 while the high potential was assumed to be 12. The units are arbitrary since the voltage ratio only has any effect on the lens properties.

The plot of the axial potential was broken up into straight segments as in figure 7. The value of z, in terms of R, and ϕ at the points of intersection of the straight segments are given in table 1.

Table 1.

	z (in terms of R)	ϕ
A	-0.31	2.00
B	0.59	2.90
C	1.02	3.85
D	1.42	5.00
E	1.70	6.30
F	1.91	7.65
G	2.04	9.55
H	2.78	11.10
I	2.95	11.40
J	3.45	12.00

The values of the slopes of the segments and thus the potential gradients, ϕ', are given in table 2.

Table 2.

Segment	ϕ'
∞A	0
AB	0.24
BC	0.54
CD	0.72
DE	1.25
EF	1.33
FG	1.11
GH	0.91
HI	0.60
IJ	0.36
J∞	0

The results of the calculations are given in table 3.

Table 3.

	r	r_1'	r_2'	c
A	0.250	0	-0.008	-0.011
B	0.221	-0.006	-0.008	-0.014
C	0.207	-0.007	-0.009	-0.018
D	0.183	-0.008	-0.013	-0.029
E	0.170	-0.012	-0.018	-0.045
F	0.154	-0.016	-0.015	-0.041
G	0.130	-0.013	-0.012	-0.037
H	0.110	-0.011	-0.010	-0.033
I	0.107	-0.010	-0.009	-0.030
J	0.093	-0.009	-0.008	

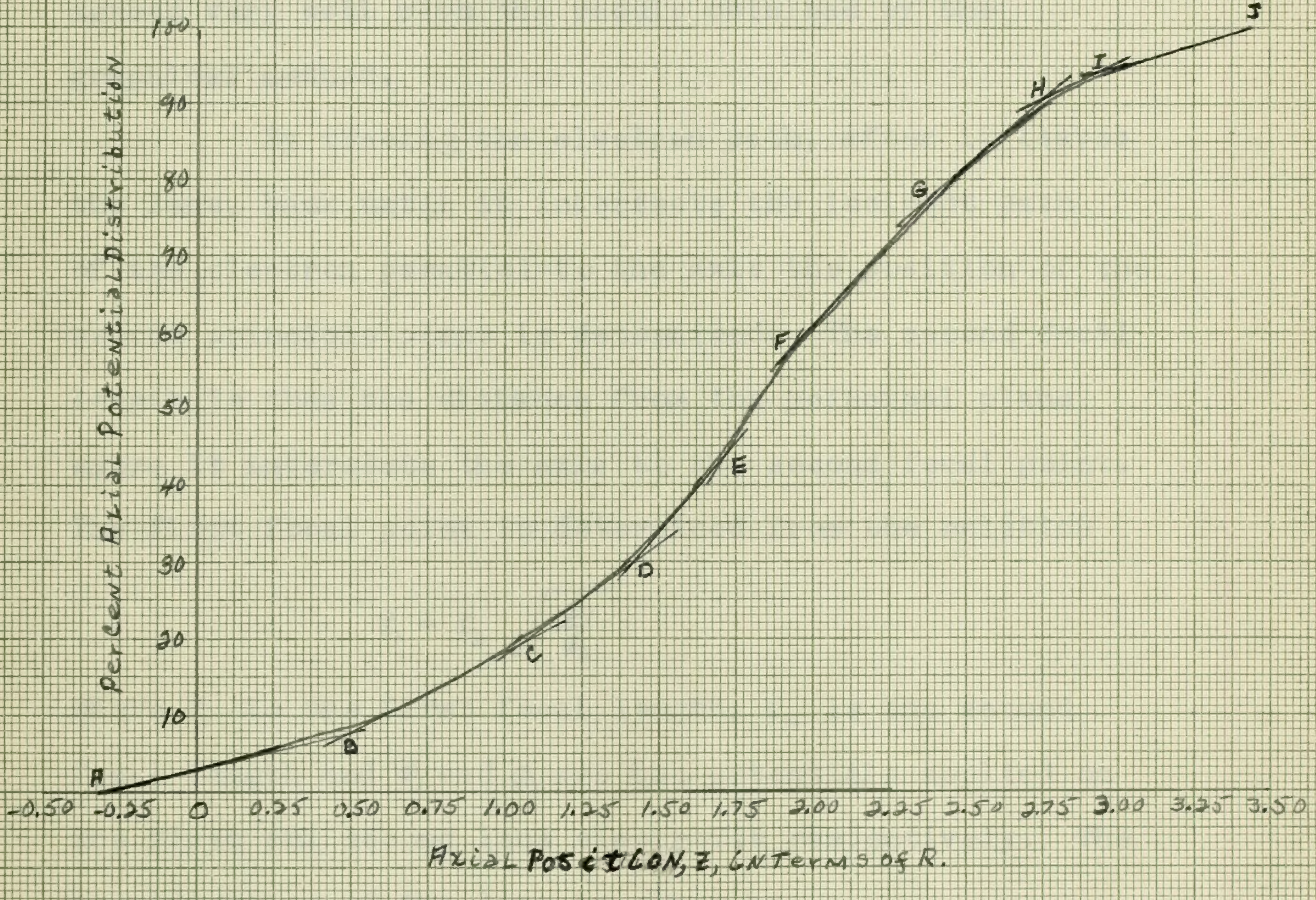
Percent Axial Potential Distribution
100
90
80
70
60
50
40
30
20
10
-0.50
-0.25
0
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
A
B
C
D
E
F
G
H
I
J
Axial Position, Z, in Terms of R.

Figure 7.

From the point J the electron continues in a straight line with a slope to the axis of -0.008. Thus it will intersect the axis at a distance 0.093/0.008 = 11.6R from the point J or a distance of 15.05R from the zero point of the axis. This value agrees very well with the result obtained by the graphical method.

By tracing the electron path, after it leaves the lens, back until it intersects the incident path of the electron parallel to the axis the position of the second principle plane is determined. The second focal length is then the distance from the principal plane where it intersects the axis to the second focal point. The first focal point is determined from the relation

$$f_1 = f_2 \sqrt{\frac{\phi_1}{\phi_2}}$$

where f_1 is the first focal length, f_2 is the second focal length and ϕ_1 and ϕ_2 are the potentials applied on the first and second lens electrodes respectively. The magnification of the lens is given by

$$m = \frac{V}{f_2} = \frac{f_1}{U}$$

where V is the distance from the second focal point to the image plane, and U is the distance from the first

focal point to the object plane. Thus once the position of the electron emitting object has been selected all of the other constants and distances in the lens can be calculated from the relationships given.

In order to protect the focussing properties of the lens from magnetic fields, the entire lens structure must be shielded with iron. Since the lens will be near strong magnetic fields, the iron shield must be heavy enough to adequately protect it from these fields. A small aperture will have to be cut in the end of the iron shield to allow the beam of electrons leaving the lens to pass into the spectrometer. This aperture will also serve to act as a shield against stray electrons in the lens structure and to cut down on the effect of spherical aberration.

The electron emitting object must be so placed, and the beam of the electrons defined by apertures in such a way, that the paths of the electrons entering the lens do not make too large an angle with the axis of the lens. The paths should not make an angle larger than such that the sine of the angle may be considered as equal to the angle in radians. This will assure a better focal point and less spherical aberration.

PART II

The Spectrometer

After passing through the aperture in the end of the iron shield around the lens system, the beam of electrons enters into a uniform magnetic field directed perpendicular to the axis of the beam. The magnetic field will now cause the electrons to follow circular paths of radius of curvature determined by the velocity and, thus the energy, of the individual electrons.

The force acting on an electron moving in a magnetic field is given by

$$F = Bev \tag{1}$$

where B is the magnetic induction, e is the charge on the electron, and v is the velocity of the electron. The force required to keep a particle of mass $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ and velocity v moving in a circular path of radius r is

$$F = \frac{mv^2}{r} \quad . \tag{2}$$

These forces must be equal if the particle is to remain in a circular path when acted on by both of these forces. Thus equating the right hand sides of equations (1) and (2) we get

$$Bev = \frac{mv^2}{r} \quad . \tag{3}$$

Solving for r we get

$$r = \frac{mv}{Be} \qquad (4)$$

which gives the radius of the path of any particle with mass m, velocity v, charge e in a magnetic field of induction B. Usually the energy is expressed in Hr or Br units. Thus multiplying both sides of equation (4) by B we get

$$Br = \frac{mv}{e} \quad . \qquad (5)$$

Since the value of B is known, the energy of any electron, in terms of Br, can be determined by measuring the radius of curvature. The angular position of the detector from the axis of the beam as it leaves the lens system will determine the radius of curvature of the electrons entering the window of the detector.

In order to get maximum intensity at the detector, it is necessary to place the window of the detector at the image plane of the beam of electrons. It is thus necessary to calculate the curve described by the image plane as the beam of electrons is bent into successively smaller circles. We take the exit

aperture from the lens system on the axis of the beam of electrons as the origin of the coordinate axes. The equation of the family of circles passing through the origin with their centers on the y-axis below the x-axis is given by

$$x^2 + (y+r)^2 = r^2 \quad . \tag{6}$$

This equation represents the paths of the electron beam as a varying magnetic field is applied. Since the distance from the origin to the image plane is a constant length regardless of the radius of curvature, the image plane will intercept an arc of equal length on each circle of the family.

The arc length S of any curve is given by

$$S = \int_a^b \left[\left(\frac{dx}{dy}\right)^2 + 1\right]^{1/2} dy \quad . \tag{7}$$

For the family of curves of equation (6)

$$\frac{dx}{dy} = -\frac{y-r}{x} \quad . \tag{8}$$

In this case a = 0. Thus

$$S = \int_0^b \left[\frac{y \quad +2yr+r}{x} \quad +1\right]^{1/2} dy \tag{9}$$

and integrating gives

$$S = r \arccos\left(1+\frac{b}{r}\right) \tag{10}$$

or

$$\cos \frac{S}{r} = 1 + \frac{b}{r} \quad . \tag{11}$$

Solving for r we get

$$r = - \frac{b}{1 - \cos(S/r)} \quad . \tag{12}$$

S is constant because the distance from the origin to the image plane is of constant length on any of the circles.

The equation of the family of circles in polar coordinates is given by

$$\rho = - 2r \sin\theta \quad . \tag{13}$$

Solving this equation for r we get

$$r = - \frac{\rho}{2\sin\theta} \quad . \tag{14}$$

We now substiture this value of r in equation (12) giving

$$- \frac{\rho}{2\sin\theta} = - \frac{b}{1-\cos\left(\frac{S}{\frac{-\rho}{2\sin\theta}}\right)} \quad . \tag{15}$$

In the integration of S, b is the value of y at the termination of the arc. Therefore we substitute

$$b = y = \rho \sin\theta \tag{16}$$

for b in equation (15). Then

$$\frac{\rho}{2\sin\theta} = \frac{\rho\sin\theta}{1-\cos\left(\frac{2S\sin\theta}{\rho}\right)} \quad , \tag{17}$$

$$\rho\cos\left(\frac{2S\sin\theta}{\rho}\right) = \rho(1-2\sin^2\theta) = \rho\cos 2\theta , \tag{18}$$

$$\rho\left[\cos\left(\frac{2S\sin\theta}{\rho}\right) - \cos 2\theta\right] = 0 \quad . \tag{19}$$

Thus one of the solutions is the trivial case where $\rho = 0$. If $\rho \neq 0$,

$$\cos\left(\frac{2S\sin\theta}{\rho}\right) = \cos 2\theta \quad , \tag{20}$$

$$\frac{2S\sin}{\rho} = 2\theta , \tag{21}$$

$$\rho = \frac{S\sin\theta}{\theta} \quad . \tag{22}$$

We can expand $\sin\theta$ in a series to avoid trouble when θ becomes zero. Then

$$\rho = \frac{S}{\theta}\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right) , \tag{23}$$

$$\rho = S\left(1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \cdots\right) \quad . \tag{24}$$

This is the equation of the curve describing the shape of the exit window.

When the value of ρ given in equation (22) is substituted for ρ in the equation of the family of circles in polar coordinates (equation (13)), r can be solved for in terms of S and θ. Thus

$$\frac{S\sin\theta}{\theta} + 2r\sin\theta = 0 \quad . \tag{25}$$

When $\sin\theta \neq 0$,

$$\frac{S}{\theta} + 2r = 0 \tag{26}$$

or

$$r = -\frac{S}{2\theta} \quad . \tag{27}$$

Since S is constant, this equation determines the radius of the path of any beam of electrons in terms of the angular displacement of the detector from the axis of the lens system.

This value of r does not determine the true energy of the particles, however, because all of the particles from the source were accelerated by the focussing lens. We will write equation (5) as

$$Br_o = \frac{mv}{e} \tag{28}$$

where Br_o indicates the observed value. The true value is then

$$Br_t = \frac{m}{e}(v - v_a) \tag{29}$$

where v_a indicates the velocity of the particles due to the acceleration of the particles in the lens.

The force $\underline{F}$ acting on an electron in an electric field $\underline{E}$ is given by

$$F = eE \quad . \tag{30}$$

If the potential is zero at the source, then

$$1/2\, mv^2 = eE \quad . \tag{31}$$

Since many of the particles are moving at relativistic speeds, it is necessary to make corrections for change in mass. Then

$$K.E. = eE = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right] \quad . \tag{32}$$

Solving for $\underline{v}$ we get

$$\left(\frac{eE}{m_0 c^2} + 1 \right)^2 = \frac{c^2}{c^2 - v^2} \quad , \tag{33}$$

$$v^2 = c^2 - \frac{c^2}{\left(\frac{eE}{m_0 c^2} + 1 \right)^2} \quad , \tag{34}$$

$$v = c \sqrt{1 - \frac{1}{\left(\frac{eE}{m_0 c^2} + 1 \right)^2}} \quad . \tag{35}$$

This is the velocity due to the acceleration produced by the electric field, v_a . Thus

$$Br_t = \frac{m}{e}\left(v - c\sqrt{1 - \frac{1}{\left(\frac{eE}{m_0c^2}+1\right)^2}}\right). \qquad (36)$$

But $Br_0 = \frac{mv}{e}$.

Therefore

$$Br_t = Br_0 - \frac{mc}{e}\sqrt{1 - \frac{1}{\left(\frac{eE}{m_0c^2}+1\right)^2}} \qquad (37)$$

We have thus determined the true energy of the particles entering the detector.

When beta-particles pass through matter, they lose energy due to the ionization produced along their paths. The number of ion pairs formed increases with decreasing velocity of the beta-particles. Some of the energy of the particles is lost due to the excitation of atoms without actually forming ion pairs. Furthermore, some of the particles will be scattered by interaction with the nuclei of the atoms. Thus the entire lens area and spectrometer must be highly evacuated to prevent loss of energy and scattering of the electrons in passing from the source to the detector.

However, since the spectrometer must be highly evacuated, an exit window must be provided through which the electrons can pass into the detector. It has been found that the average angle of deflection of electrons in passing through thin layers of matter is proportional to the square root of the thickness of the matter[1] and that the scattering at a particular angle is proportional to the square of the atomic number of the material.[2] Thus in selecting the material for the exit window, consideration must be given to the atomic number, ionization potential, and strength of the material. The atomic number and ionization potential should be low, and it must be possible to obtain a very thin layer which will have sufficient strength to withstand the atmospheric pressure when the spectrometer is evacuated. Lawson and Tyler[3] have used pyroxilin in the exit window of their magnetic semi-circular focussing spectrometer. The pyroxilin is dissolved in dry amyl acetate and poured on a tank of water. The layer is spread out as thin as desired, and then a wire frame is brought up from the bottom of the layer so that the pyroxilin layer hangs over the wire. If the layer so produced is too thin, the process is

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[1] J.A.Crowther,Proc.Roy.Soc. A,84,226(1910).
[2] J.A.Crowther and B.F.Schonland,Proc.Roy.Soc,A,100,526 (1922).
[3] J.L.Lawson and A.W.Tyler, Rev.Sci.Inst.,11,6,(1940).

repeated with another layer until the desired thickness is obtained. Films as thin as 1 1/2 microns were used that withstood pressures of 30 cm. of mercury over a rectangle of 4 m.m. by 16 m.m. for extended periods of time.

By making the film slightly thicker and placing a support at each 16 m.m. interval it will be possible to construct an exit window large enough so that two detectors can be used simultaneously at the exit window thus making it possible to make coincidence counts between various homogeneous groups of electrons focussed simultaneously from the same source. This would give much information on the problem of beta-disintegration.

CONCLUSIONS

The beta-ray spectrum extends from zero energy up to a certain maximum which depends on the nuclei of the atom which is emitting the beta-particles. The low energy particles in the spectrum cannot be studied very accurately with most spectrometers in use today because of the fact that these particles are very easily stopped by the exit window. The higher the atomic number and thickness of the material, the greater the stopping power will be. Thus many investigators have attempted to improve the results found in the lower end of the spectrum by devising exit windows and counter windows that are thinner and of a lower atomic number.

The spectrometer proposed in this paper would overcome these difficulties encountered in the lower end of the spectrum by the fact that all of the particles are accelerated by the electrostatic lens. This acceleration would give the low energy particles sufficient energy to be able to pass through the exit window. The amount of acceleration of the particles can be increased or decreased as desired by increasing

or decreasing the potential difference between the lens electrodes without changing the focussing properties of the lens because these properties depend only on the voltage ratio. The thickness of the exit window would then not be so critical although it would still be important.

Since the thickness of the exit window would now not be so critical, it would be possible to make the window larger so that two detectors could be used simultaneously. It would then be possible to make coincidence counts between homogeneous groups of electrons of different energies focussed from the same source. So far as is known, it has been impossible to accomplish this with any spectrometer in use at the present time. Much valuable information concerning beta-disintegration could be obtained by this scheme.

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ABSTRACT

The purpose of this paper is to develop the theory of a new type of beta-ray spectrometer utilizing the focussing of the electrons by an electrostatic lens.

The theory of electrostatic lenses is discussed in Section A of Part I. In order to calculate the path of an electron through the lens it is first necessary to determine the potential distribution by the solution of the Laplace equation with boundary conditions depending on the shapes of the electrodes and the potentials applied.

The general solution of the Laplace equation in cylindrical coordinates is worked out, and specific solutions for the case of two coaxial cylinders of equal diameters are given for the axial potential distribution and its first and second derivatives. These solutions must be evaluated by approximation schemes to obtain numerical values of the potential which can then be used to calculate the electron path through the lens.

With a more complicated set of electrodes, as was necessary in this case, an analytic solution of the Laplace equation becomes difficult in the extreme or impossible. Experimental methods must then be employed.

The method of tracing an electron through the lens analytically is discussed. From the results it is shown that the path of the electron depends only on the geometry of the field and on the ratio of the potentials applied to the electrodes.

The experimental procedures employed in the determination of the potential distribution of the lens and the results obtained are given in Section B of Part I. The potential distribution was plotted by means of an electrolytic plotting tank. Since the current flowing between the electrodes in the electrolyte acts like an incompressible fluid, the current density and thus the field strength obey the equation of continuity, or the potential throughout the electrolyte obeys the Laplace equation. The potential in the liquid is then proportional to the potential in the actual lens system.

The models of the electrodes used in the plotting tank were large replicas of the actual electrodes cut through a plane of symmetry. The shapes of the electrodes were determined by the fact that the distances between any two successive equipotential surfaces measured along the field vector should be the same over a considerable portion of the lens.

When a potential difference was applied to the electrode models in the electrolyte, a probe which was set at a percentage of the potential difference was moved in such a way that the current to it remained zero as determined by a null detector. The probe was connected to one end of a pantograph linkage, the other end of which was provided with a pencil where the motion of the probe was recorded on graph paper. The position of the equipotential surfaces in terms of percentage of the potential difference is plotted for values from five percent to ninety-five percent.

The axial distribution is plotted as a function of the axial position which is measured in terms of the diameter of the smaller aperture of the electrodes. The field strength is then determined by plotting the slopes of the curve of axial potential against the axial position. The total field strength is determined by finding the area under the field strength curve.

The path of an electron is traced through the lens by two methods, a graphical method and an analytic approximation method. The graphical scheme is the parabola method which is based on the fact that the tangent to a parabola at a point distant x from its vertex meets the axis at a distance 2x from the

point of tangency, and that an electron moving in a uniform field with a velocity component at right angles to the field will follow a parabolic path.

The analytic approximation method consists of representing the axial potential plot as a series of straight line segments and then applying the ray equation along these segments in turn. The results of the two methods are shown in tables and graphs, and they check very favorably.

From the results of the ray tracing the position of the second focal point and then the second principal plane are found. All of the optical characteristics of the lens can then be determined from optical relationships between these characteristics.

The actual spectrometer is discussed in Part II. The entire lens portion is to be encased with an iron shield to prevent magnetic fields from interfering with the focussing properties. The beam of electrons can enter the magnetic field of the spectrometer through a small aperture in the iron shield.

In order that the image plane shall always be located at the exit window regardless of how much the beam electrons is bent by the magnetic field the shape of the exit window relative to the position of

the axis of the beam of electrons leaving the aperture in the iron shield is calculated. From this relationship the radius of curvature of the path of any electron reaching the detector at the exit window can be determined in terms of the angular displacement of the detector from the axis of the beam leaving the lens.

The true radius of curvature of the path of any electron is calculated by taking into account the amount of acceleration produced by the electrostatic lens on the electrons and by considering relativity effects.

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